EVALUATING EVOLUTIONARY MECHANISMS BY SIMULATION

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ABSTRACT

A three year research project is investigating evolutionary processes in electronic markets. Three fundamental evolutionary mechanisms are innovation, imitation and improvement of existing procedures. As part of the initial investigation, simulation experiments have been performed to investigate the relative impact and cost of these three evolutionary mechanisms. The design of those simulation experiments is described here together with a description of the implementation and some preliminary results. The simulation was implemented in Java and is available over the Internet as an applet. The experiments are based on a relative demand function that peaks early and then tends to zero over time. So the markets to which these results apply are those in which fashion and 'fad' are significant factors, such as the market for electronic goods.

KEY WORDS
dynamic systems, economic simulation, evolution

1. Introduction

A three year research project commenced in 2002 at UTS to investigate the evolutionary processes in electronic markets. The project is funded by the Australian Research Council. Three fundamental evolutionary mechanisms are innovation, imitation and improvement of existing procedures. As part of the initial investigation, simulation experiments have been performed to investigate the relative impact and cost of these three evolutionary mechanisms [1]. The design of those simulation experiments is described here together with a description of the implementation and a summary of preliminary results.

The simulation experiments described here are in simulation of dynamic economic systems. The basis for this investigation is in the tradition of Nelson and Winter [2]. The approach taken owes much to [3]. A major consideration in designing simulation systems of economies is keeping things simple; otherwise the system may have so many variables that determining suitable values for the basic parameters becomes so complex that the results of the simulation are meaningless. To keep our model simple we have worked within a closed economy containing a fixed amount of money, with a fixed number of workers (the "labour", where labour is also the market. That is, labour manufactures goods (or "output") for a time, are paid and then purchase the output that they have produced with the money that they have been paid.

2. The Basis for the Investigation

The model described here is designed to enable the performance of firms to be compared when they allocate various proportions of their labour to: producing output (workers), discovering new outputs (innovators), copying outputs of other firms (imitators), and improving the efficiency of production (process improvers). These comparisons are conducted in a "closed economy"—that is, a trading environment in which a fixed total amount of labour (ie: employees) is paid by firms for generating output, and the employees purchase this output using all of the money that they have been paid. All of this takes place in successive, discrete time periods. At the beginning of each time period each firm has a budget for its labour. Each firm hires labour to the full extent of its labour budget. In the "final few moments" of each time period the following things happen:

- labour is paid by the firms in exchange for their work during that time period—at this stage labour has all the money and the firms have none;
- the firms are paid by labour in exchange for the output—all output is either sold or written off before the next time period starts—at this stage the firms have all the money and labour has none;
- the firms are now "cashed up" and they commit all of their money by hiring labour for the next time period.

If a firm receives no income in a particular time period then it will have spent all of its budget on hiring labour for that time period, will have nothing left for the next time period, and so it will go out of business. A firm's *profit* in a time period is the amount that it receives for selling its output at the end of that time period less the amount that it spent on hiring labour at the beginning of that time period. If a firm makes a profit during a time period then its budget is increased in the next time period and so it will hire more labour than in the previous time period. If it makes a loss then its budget is decreased and size of its labour force contracts in the next time period. The objective of each firm is to survive [4]. The total amount of money in the economy remains constant in time and is all placed on the table at the end of each time period as described above. The size of the labour force also remains constant as does the total and per capita remuneration that labour receives. At the beginning of each time period all money is committed by firms to hiring labour.

The firms differ in the way in which they allocate their money at the beginning of each time period to four distinct types of job. The four job types are:

- *workers* who produce output—the proportion of firm i's money spent on workers is w_i.
- *process improvers* who improve work processes by generating "process knowledge"—that is knowledge of how to produce output better—the proportion of firm i's money spent on process improvers is p_i.
- *imitators* who design processes for producing outputs that have been discovered by other firms—the proportion of firm i's money spent on imitators is m_i.
- *innovators* who discover new outputs—the proportion of firm i's money spent on innovators is n_i.

If a firm discovers a new output during a time period then, at the end of that time period, other firms may decide to attempt to copy that output.

The objective of the simulation experiments described here is to understand the effect of values for the four basic variables w_i, p_i, m_i and n_i on a firm's performance. These variables are constrained by:

$$0 \le \{w_i, p_i, m_i, n_i\} \le 1$$
$$w_i + p_i + m_i + n_i = 1$$

for $i = 1,..,n$ where n is the number of firms.

3. The Structure of the Investigation

The basic structure of the model, from the point of view of the economy, is shown in Figure 1. It owes much to [3]. At the beginning of each time period a labour force of fixed size is fully employed by a number of firms at a fixed wage rate. During each time period, the total costs for each firm are the amount it spends on hiring labour. The total costs for firm i are C_i. The total costs for all firms is $\Sigma_i C_i$, and this amount of money is entirely spent on hiring labour and so this is also the amount of money that the entire labour force will spend at the end of the time period when they purchase output. In each time period firm i allocates the effort of its workers across the range of outputs that firm i knows how to produce. That allocation of workers will lead—as determined by each output's process knowledge—to the generation of actual output $\underline{Q}_i$ for firm i—where the underlining notation denotes a vector $\underline{Q}_i = [q_{i,1}, q_{i,2}, q_{i,3},....]$—that is, $q_{i,j}$ is amount of the j'th output that firm i produces in the time period. The total quantity of the j'th output that is available at the end of the time period is $\Sigma_i q_{i,j} = q_i$. The total output, produced by all firms, at the end of the time period is represented as the vector $\underline{\mathbf{Q}} = [q_1, q_2, q_3,...]$. The price of the various types of output is determined so that the total cost of all outputs is exactly the same as the amount of money that labour has to spend. That is, price is set to ensure that supply equals demand. At the end of the time period the entire labour force "goes shopping" and purchases all of the output $\underline{\mathbf{Q}}$. The $\underline{\mathbf{Q}}$ vector is unbounded in length although at any time only a finite number of entries in it will be non-zero.

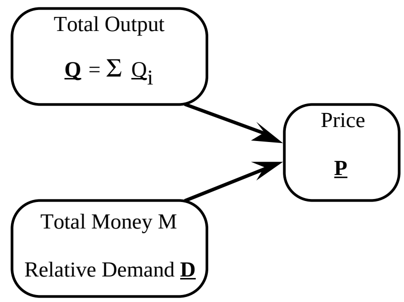

Figure 1. The model from the point of view of the economy.

Having determined the price vector $\underline{\mathbf{P}}$, the model from the point of view of firm i is shown in Figure 2. Consider the time period [t – 1, t]. At the beginning of this previous time period the firm will have carried over its revenue R_i^{t-2} derived in the previous time period and will have fully committed this revenue to hiring labour. The way in which the output vector $\underline{Q}_i^{t-1}$ and the costs C_i^{t-1} are determined for the output produced *during* the time period [t – 1, t] is described below in Figure 3. Having determined the output vector, and having calculated the price vector $\underline{P}_i^{t-1}$ so as to clear the market as described above, the revenue for firm i, which is derived at the end of the time period [t – 1, t], is:

$$R_i^{t-1} = (p_1 \times q_{i,1}) + (p_2 \times q_{i,2}) + \quad = \Sigma_j (p_j \times q_{i,j})$$

Hence the profit for this time period, S_i^{t-1}, is determined and so is the revenue that will be carried over to the next time period. The "anti-clockwise loop" shown in Figure 2 goes "round and round" from one time period to the next.

Figure 2 does not show how the carry over amount R_i^{t-2}, available at the start of time period [t – 1, t], generates output $\underline{Q}_i^{t-1}$ and costs C_i^{t-1} by the end of that time period. This is shown in Figure 3. The horizontal dashed line in Figure 3 divides the figure into two time periods: [t–2, t–1] in the upper part, and [t–1, t] in the lower part. First, the carry over amount R_i^{t-2} from [t–2, t–1] becomes the budget for the time period [t–1, t]. The budget R_i^{t-2} is entirely committed to hiring labour in the time period [t–1, t]. That is:

$$L_i^{t-1} = \frac{R_i^{t-2}}{c}$$

where c is the constant wage rate. For simplicity, c is set to unity. So a "unit of money" is the cost of a unit of labour for one time period. Labour is split in the proportions $w_i : p_i : m_i : n_i$ into the four categories workers, process improvers, imitators and innovators. The imitators attempt to build processes for producing outputs that have been discovered by other firms. If they are successful then they create a level of manufacturing expertise, or process knowledge, that is represented as a vector:

$$\underline{Imi}_i^{t-2}$$

For example:

$$\underline{Imi}_i^{t-2} = [0, 0, 1.0, 0, 0, 0, 0, 0, ...]$$

contains process knowledge with value 1.0 concerning output 3. The value 1.0 is added to the third place of the firm's process knowledge vector—this is described below. The value of a firm's process knowledge for an output will be 0.0 if the firm can not produce that output, and 1.0 if it has discovered how to produce that output by either innovation or imitation. The value of the process knowledge may then be increased to an integer value greater than 1.0 by the firms process improvers. The process improvers generate process knowledge for outputs that the firm already produces.

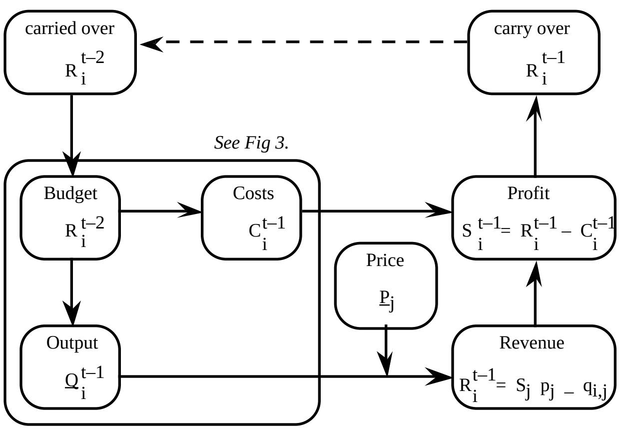

Figure 2. The model from the point of view of a particular firm i.

A firm's process improvers are allocated to improving the manufacturing processes for particular outputs. The i'th firms *process knowledge* is denoted by a vector $\underline{A}_i$. In the time period $[t-1, t]$ the process improvers may have found new process knowledge $\underline{Pro}_i^{t-1}$ this knowledge is represented as a vector denoting the output(s) that are the subject of the generated process knowledge. Likewise the innovators $\underline{N}_i$ may discover process knowledge for new outputs, $\underline{Inno}_i^{t-1}$. All knowledge generated during one time period may only be used in subsequent time periods, and so each firms process knowledge available in the period $[t-1, t]$ is:

$$\underline{A}_i^{t-1} = \underline{A}_i^{t-2} + \underline{Imi}_i^{t-2} + \underline{Pro}_i^{t-2} + \underline{Inno}_i^{t-2}$$

That is, each firms process knowledge accumulates from one time to the next. It remains to describe how a firm's workers use this knowledge. Firm i's workers are distributed across the range of outputs that the firm can produce as represented by the vector $\underline{W}_i^{t-1}$. The quantity of output that the workers generate in the time period is:

$$\underline{Q}_i^{t-1} = \underline{A}_i^{t-1} \underline{\times} \underline{W}_i^{t-1}$$

where the $\underline{\times}$ symbol means that the vectors are multiplied together element by element.

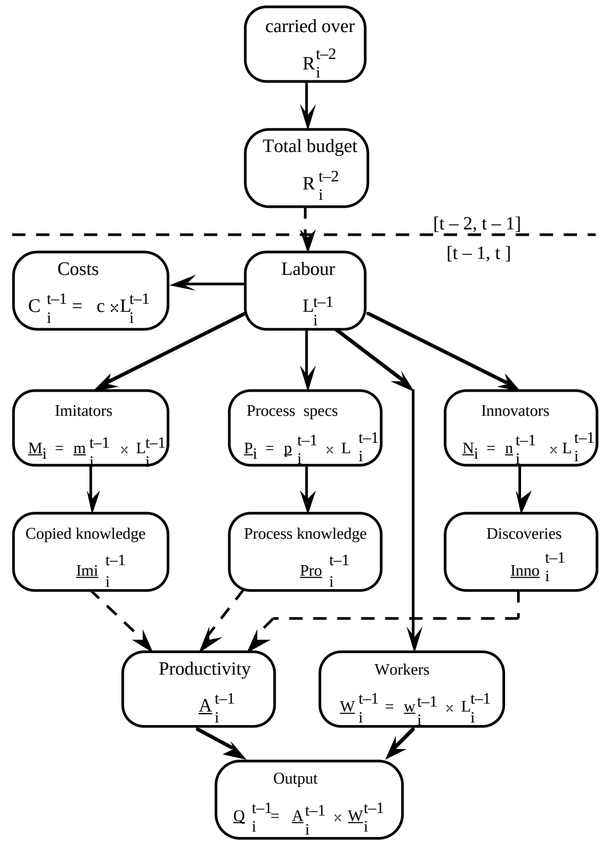

Figure 3. An allocation of resources leads to output and costs for firm i. The dashed line separate two time periods, and dashed arrows mean that the new knowledge is not available until the following time period.

4. Determining demand

The price of each type of output is determined at the end of each time period by the amount of output generated in that time period, by the total amount of money available, and by the "relative demand" for the different types of output which is determined by labour's preferences. Relative demand reflects the preferences of labour for different types of output. So a model of relative demand for each output is required to calculate unit price, as is a model of supply—ie: the output generated. Relative demand is considered now, and supply is considered in the next sub-section.

A model of relative demand is derived by considering the development of demand for an output in a market with fixed total demand. In practice the development of demand will depend on the type of product; the development of the demand for automobiles may not be the same as the development of demand for a new beverage because at best each member of the population will purchase one or two automobiles but may purchase a beverage repeatedly. The type of product chosen here is outputs such as packaged "complete dinners" in a market of fixed total demand. [This example should not be taken as an indication of the gastronomic preferences of the author.] That is, in each time period there is a total demand for a fixed σ units of output (eg: σ packaged dinners). σ is called the *market size*. Given a particular output (eg; a particular packaged dinner), in a particular time period $[t-1, t]$, the *initial penetration*, P^{t-1}, is the size of the population who has purchased this output at least once either during or before this time period. In time period $[t-1, t]$, the *first-time sales*, N^{t-1}, are sales made of this output during this period to those who have not purchased this output previously. Suppose that the growth of initial penetration t is proportional, for some *penetration constant* γ, to the size of the population that has yet to purchase this output. Then initial penetration in time period $[t-1, t]$, P^i, satisfies:

$$P^0 = \gamma \times \mu$$
$$P^1 - P^0 = \gamma \times (\mu - P^0)$$
$$P^2 - P^1 = \gamma \times (\mu - P^1)$$

Or as a continuous approximation:

$$\frac{dP}{dt} = \gamma \times [\, \mu - P \,]$$

Solving this differential equation gives the initial penetration:

$$P^t = \mu \times (1 - \exp(-\gamma\, t\,))$$

First-time sales is the rate of change of initial penetration. So if N^t is first time sales at time t:

$$N^t = P^t - P^{t-1}$$

and as a continuous approximation:

$$N^t = \frac{dP^t}{dt} = \mu \times \gamma \times \exp(-\gamma\, t\,)) \qquad (1)$$

First-time sales for a market of size $\sigma = 100$ and penetration constant 0.1 is shown in Figure 4.

Now suppose that once labour has purchased an output, labour continues to purchase that output with a probability of α. That is, if T^i is total sales in time period $[i-1, i]$:

$$T^{i+1} = N^{i+1} + \alpha \times T^i$$

where N^i is first time sales in time period $[i-1, i]$. Then:

$$T^0 = N^0$$
$$T^1 = \alpha \times N^0 + N^1$$
$$T^2 = \alpha^2 \times N^0 + \alpha \times N^1 + N^2$$

etc

Or as a continuous approximation:

$$T^t = \int_{i=0}^{t} \alpha^{t-i} \times N^i \times di$$

Evaluating this using equation (1):

$$T^t = \frac{\mu \times \gamma}{\ln(\alpha) + \gamma} \times \left[\, \alpha^t - \exp(-t \times \gamma) \,\right] \qquad (2)$$

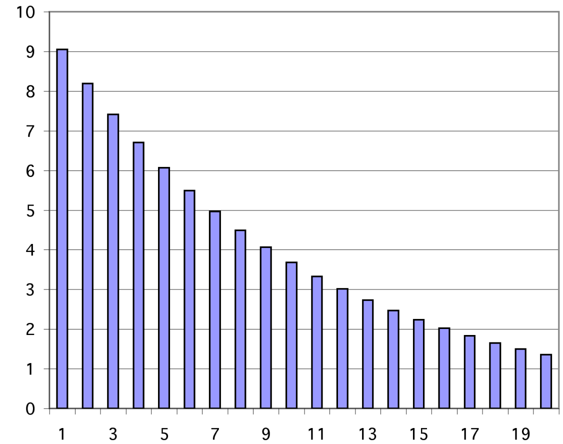

Figure 4. First-time sales, N^t, for a market of size 100 and $\gamma = 0.1$.

Which, for a market size of $\sigma = 100$ gives total sales values for each time period as shown in Figure 5 for various γ and α. The sales graphs in Figure 5 are now used to model *relative demand*. The discovery of a new output by an innovating firm leads to a substantial perturbation of the model if the graph climbs "too high too quickly" such as the graph with $\gamma = 0.2$ and $\alpha = 0.9$, which penetrates nearly 50% of the market within 8 time periods. The choice of α and γ in the simulations described below substantially effects the performance of the model—see Figure 5.

For a given market size σ, equation (2) has two variables: α and γ. Given the values of a total sales function in the first two time periods, f^0 and f^1, it is easy to calculate α and γ:

$$\gamma = \frac{f^0}{\sigma}$$
$$\alpha = \frac{f^1}{f^0} - \frac{\sigma - f^0}{\sigma}$$

and so knowing the first two values of a total sales function is to know "all there is" about it.

Returning now to the problem of modelling relative demand. The general shape of the total sales function in Figure 5 is a fair description of how interest in a new output, such as packaged foodstuffs, might be expected to develop. Equation (2), for some values of α and γ is used here to model relative demand. So each output has a relative demand determined by equation (2), with its own values of γ and α, for some fixed arbitrary σ, say, $\sigma = 1$. Labour distributes its money over the different outputs in proportion to their relative demand **D** for each output as described above. The prices per unit of the outputs is in proportion to their relative demand, and are set so as to clear the market.

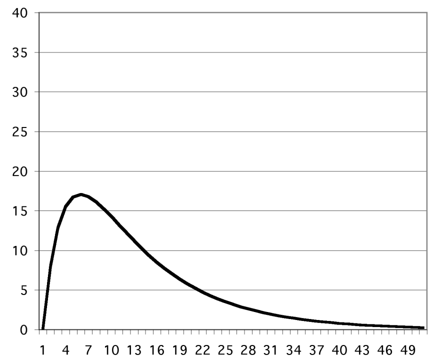

Figure 5. Total sales for each time period for a market of size μ = 100, γ = 0.1 and α = 0.7.

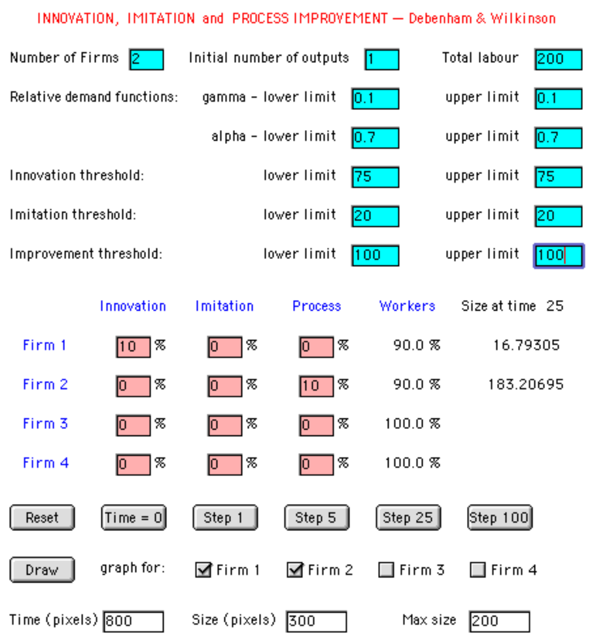

Figure 6. The Java applet.

5. Implementation

The "economy" described above has been implemented as a Java applet and is available on the World Wide Web at:
`http://www-staff.it.uts.edu.au/`
`~debenham/research/evolution1/`
The use of Microsoft Internet Explorer with Java enabled is recommended.

The applet window is in three parts—see Figure 6:

- a "blue boxes" in the top section in which the basic parameters are set.
- a "pink boxes" in the middle in which each firm's labour is assigned.
- a "white boxes" at the bottom that controls the graphical presentation.

The general idea is that the blue boxes in the top half of the applet should be set before the system is run and may not be changed without initialising the system—ie: by setting time back to zero. On the other hand the values in the pink boxes may be changed during a run—this enables a firm's labour deployment strategy to be modified as things progress [5]. When a run commences, all of the firms in the simulation produce the same number of outputs that are unique to each firm. This initial number of outputs is set in the top row. The specification of γ, α, innovation threshold, imitation threshold and improvement threshold are given as ranges. If, for example, the range for γ is set to [0.1, 0.1] then $\gamma = 0.1$. If the range is set to [0.1, 0.2] then γ will be set to a random number in this range. The random distribution used is a truncated normal distribution—that is, quantities more than 2 standard deviations from the mean are discarded. The left-hand button "Reset" sets the values in the pink boxes to zero. The "Time = 0" button should be used to initialise the system before a run. The four buttons "Step 1",.., "Step 100" move time forward by the designated amount and the resulting sizes of the firms should appear on the lower right-side of the applet window. Once the program has been run, the "Draw" button should open another window with a graph of the resulting firm sizes. The three "white" text boxes at the bottom of the applet window may be used to control the size and proportions of the graphical output.

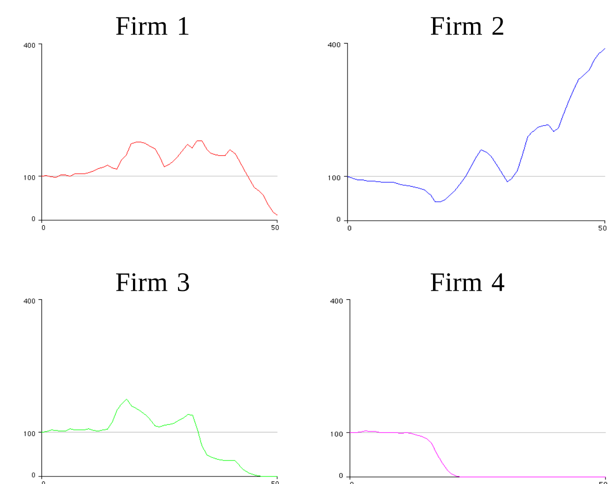

Figure 7. Size of four firms when: the first allocates 6% to imitation and 6% to process improvement, the second allocates 6% to innovation and 6% to process improvement, the third allocates 6% to innovation and 6% to imitation, and the fourth allocates 4% to each of innovation, imitation and process improvement. There is no randomisation of the parameters: innovation threshold = 80, imitation threshold = 12, improvement threshold = 85, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

The interplay between imitation, innovation and improvement is quite complex. Suppose that there are four firms who are prepared to allocate 12% to other than workers. Suppose that:

- the first allocates 6% to imitation and 6% to process improvement

- the second allocates 6% to innovation and 6% to process improvement
- the third allocates 6% to innovation and 6% to imitation
- the fourth allocates 4% to each of innovation, imitation and process improvement

There is no randomisation of the parameters: innovation threshold = 80, imitation threshold = 12, improvement threshold = 85, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$. The sizes of the four firms are shown in Figure 7. The fourth firm with the mixed strategy across innovation, imitation and improvement performs the worst [6].

Innovation threshold	Imitation threshold
10.0	9.6762085
15.0	10.123603
20.0	9.366529
25.0	8.693429
30.0	7.92837
35.0	7.215324
40.0	6.624544
45.0	6.3362412
50.0	5.8482556
55.0	4.824529
60.0	4.6543684
65.0	4.5592747
70.0	4.454127
75.0	4.164027

Table 1. Optimal values of the imitation threshold for values of the innovation threshold between 10 and 75 where two firms invest 5% in innovation and imitation respectively, and $\gamma = 0.1$ and $\alpha = 0.7$.

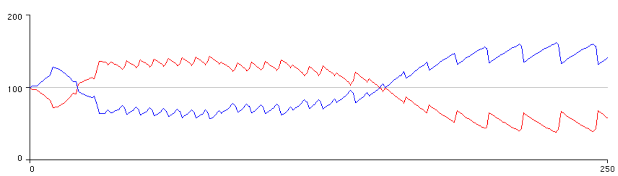

Figure 8. Two firms compete. An innovating firm allocates 5% to innovation with a threshold 40, an imitating firm allocates 5% to imitation with a threshold at 6.624544. There is no randomisation of the parameters: $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

Consider two firms the first allocates 5% to innovation and the second 5% to imitation. Given a value of the innovation threshold, what vales for the imitation threshold lead to stable performance [7]? That is, under what circumstances can an imitating firm "live off" an innovating firm? The result may at first appear counter-intuitive in that as the innovation threshold *increases* the stable imitation threshold *decreases*. The reason for this is that in a stable configuration the imitation threshold should be at a level so that the imitating firm discovers how to imitate in good time but not too soon. If it discovers an imitation early in the innovation cycle then it will have nothing else to imitate and so will allocate all of its labour to workers and so may kill the firm from which it derives its inspiration. Further the greater the innovation threshold the longer the time between innovation discoveries, the greater the relative demand of the discovered output, and the sooner the imitator must learn to imitate the output [8]. Table 1 shows sample values and Figure 8 shows the respective sizes of the firms—the imitating firm dominates.

6. Conclusion

The basis of the simulation seems sound in that the general performance of the system "sits comfortably" with expectation. The results of the experiments may only be applied directly to markets in which the relative demand functions are of the general shape used here. Preliminary indications are that completely mixed strategies of innovation, imitation and improvement are inferior to concentrated evolution strategies [9]. Given an innovating firm, an imitating firm can "live off it" with a modest investment in imitation.

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